Pair-Distribution Functions of Composite Fermion Liquids

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Abstract. Pair-distribution functions $g(r)$ of Laughlin quasielectrons (QE’s) are calculated for the fractional quantum Hall liquids at electron filling factors $\nu = 4/11$ and $3/8$. They all have a shoulder at a medium range. The intra- and inter-cluster contributions to $g(r)$ are identified, supporting the idea of cluster formation (QE pairs at $\nu = 4/11$ and QE triplets at $\nu = 3/8$).

Keywords: composite fermion, fractional quantum Hall effect, pair-distribution function
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INTRODUCTION

Pan et al. [1] recently observed the fractional quantum Hall (FQH) effect in a spin-polarized two-dimensional electron gas (2DEG) at new electron filling factors $\nu = 4/11$ and $3/8$. These values correspond to $V_{\text{QE}} = \frac{1}{4}, \frac{1}{3}$, and $\frac{1}{2}$ of Laughlin quasielectrons (QE’s), respectively. In the composite fermion (CF) model [2], each QE corresponds to a particle in the second CF LL. Pan’s discovery implies that the CF’s (QE’s) can also form incompressible states when partially filling a LL. This could not be predicted by a simple analogy with known fractional electron liquids (Laughlin [3], Jain [2], or Moore–Read [4] states), because of a different form of the QE–QE interaction [5].

From numerical diagonalization on a sphere [6] we have obtained the energy spectra and wavefunctions of up to 14 interacting QE’s. We identified the series of finite-size liquid ground states with a gap, which extrapolate to the experimentally observed incompressible FQH states. In these states, we calculated QE–QE pair-distribution functions $g(r)$. They increase as $\sim r^2$ at short range and have a pronounced shoulder at a medium range. This behavior supports the idea of QE cluster formation, suggested earlier [7] from the analysis of QE–QE interaction pseudopotential. The $g(r)$ is decomposed into short- and long-range contributions, interpreted as correlations between the QE’s belonging to the same or to different clusters. The inter-cluster QE–QE correlations are nearly the same in all three $V_{\text{QE}} = \frac{1}{4}, \frac{1}{3}$, and $\frac{1}{2}$ states, but the cluster size depends on $\nu$. That QE’s seem to form pairs at $V_{\text{QE}} = \frac{1}{4}$ and triplets at $V_{\text{QE}} = \frac{1}{2}$.

NUMERICAL DIAGONALIZATION

Following Haldane [6] we consider $N$ particles of charge $q$ on a spherical surface of radius $R$. Dirac monopole of strength $2Q$ placed in the center of the sphere is the source of magnetic field $B$, and $2Q \phi_0 = 4\pi R^2 B$. Here $\phi_0 = h/e$ is the elementary flux. Using the definition of the magnetic length, $\lambda = \sqrt{\hbar c/qB}$, this can be written as $\lambda^2 = R^2$. Here, $\lambda$ denotes the QE magnetic length corresponding to the fractional QE charge of $q = -e/3$. The lowest Landau level (LL) has angular momentum $l = Q$ and degeneracy $\Gamma = 2l + 1$.

By diagonalizing the interaction Hamiltonian, we obtain energy as a function of total angular momentum $L$. We neglect scattering between CF-LL’s and include only the second, partially filled CF-LL. Interaction in this shell is given by the QE–QE pseudopotential $V_{\text{QE}}(\mathcal{R})$, where $\mathcal{R} = 2l - L$ is the relative pair angular momentum. $V_{\text{QE}}$ is small at $\mathcal{R} = 1$ and large at $\mathcal{R} = 3$ [5, 8], in contrast to $e-e$ pseudopotential in the lowest LL, decreasing superlinearly as a function of $\mathcal{R}$. This difference precludes Laughlin correlations among QE’s.

Infinite incompressible states are represented on a sphere by combinations of $N$ and $\Gamma$ for which ground state has $L = 0$ and a significant gap. They form series with $2l = N/\nu - \gamma$, with a constant shift $\gamma$. E.g., Laughlin $\nu = \frac{1}{3}$ states occur for $2l = 3N - 3$. For QE’s, we find incompressible sequences at $2l = 3N - 7$ and $2N - 3$, corresponding to $V_{\text{QE}} = \frac{1}{4}$ and $\frac{1}{2}$, i.e., to $\nu = \frac{1}{11}$ and $\frac{1}{3}$.

PAIR-DISTRIBUTION FUNCTIONS

Pair-distribution functions $g(r)$ were calculated as the expectation values of $\hat{g}(r) = (2/N)^2 \delta(R\mathbf{\theta} - r)$ for the series of incompressible $N$-QE ground states. Here, $\theta$ is the relative angle on a sphere and $r$ measures distance along the surface. Denoting infinitesimal area by $ds = 2\pi R^2 d(\cos \theta)$ or (in magnetic units) by $ds = dS/(2\pi \lambda^2)$, we get a normalization condition in large systems, $\int [1 - g(r)] ds = 2l/N \rightarrow v^{-1}$. Since $ds = dS/(\cos \theta)$, a “local filling factor” can also be defined as $v(r) = dN/ds = (N/2l) g(r)$, satisfying $v(\infty) = v$ and $\int v(r) ds = N - 1$. 

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cluster QE–QE correlations we decompose cluster formation [7, 10, 11]. To find inter- and intra-cluster correlations do not affect local filling factor at short range, so that \( \nu(r) \approx \nu_{K}(r) \) for small \( r \). Here, \( \nu_{K}(r) \) describes a single \( K \)-cluster, or the \( K \)-particle state with the maximum total angular momentum \( L = K! - \frac{1}{2}K(K-1) \). We have calculated \( g_{K}(r) \) and then prefactors \( \beta_{K} = \lim_{r \to 0} \nu_{K}(r)/g_{0}(r) \) for different \( (K,2l) \). For example, for \( 2l = 25 \), \( \beta_{2} = 0.2768 \) and \( \beta_{3} = 0.4196 \). These values are to be compared with \( \beta = (N/2l) \alpha \) obtained for the incompressible \( N \)-QE systems.

The assumption of cluster independence is only an approximation. To test it we use a paired Moore–Read state [4, 13]. For \( N = 14 \) and \( 2l = 25 \) we got \( \beta_{MR} = 0.336 \), somewhat larger than \( \beta_{2} \). This suggests that \( \beta_{K} \) underestimates \( \beta \) in a many-body \( K \)-clustered state.

For the QEs, \( \beta \approx 0.319 \approx \beta_{MR} \) at \( \nu_{QE} = \frac{1}{5} \) \( (N = 14 \) and \( 2l = 25) \). These values suggest that QE’s form pairs and triplets at \( \nu_{QE} = \frac{1}{5} \) and \( \frac{1}{3} \), respectively.

**CONCLUSION**

We studied QE–QE pair-distribution functions \( g(r) \) of new FQH states at \( \nu = \frac{1}{11} \) and \( \frac{1}{8} \). They differ from these known for electrons at \( \nu = \frac{1}{11} \) or \( \frac{1}{8} \) by having a shoulder at \( r \approx 2.5\lambda \). Short- and long-range contribution to \( g(r) \), describing intra- and inter-cluster correlations among the QE’s were found. The results support the idea of QE clustering (into pairs at \( \nu = \frac{1}{11} \) or triplets at \( \nu = \frac{1}{8} \)).

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